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Synopsis for Ph.D and Registration

Faculty of Science (Mathematics)

Title of the Synopsis: Mathematical Analysis of Complex Epidemic Systems Incorporating Behavioral and Saturated Incidence Functions

Work plan:

1- Introduction

Mathematical modeling has become a central tool for understanding the spread, control, and long-term behaviour of infectious diseases. Traditional epidemic models often rely on simple linear or bilinear incidence functions of the form βSI, where the disease transmission rate increases proportionally with population density and number of contacts. Although mathematically convenient, such formulations rarely capture the complexities of real-world epidemics. Empirical evidence demonstrates that human behavior significantly changes during disease outbreaks: individuals adopt precautionary measures such as mask use, social distancing, or reduced mobility as infection levels rise. This behavioral adaptation leads to a *nonlinear reduction* in effective contact rates, which cannot be modeled through simple mass-action incidence.

Similarly, transmission does not increase indefinitely with population size. At high levels of infection or contact intensity, the interaction capacity saturates because individuals cannot make infinitely many contacts, healthcare resources become limited, and social structures impose natural constraints. These limitations give rise to saturated incidence functions, which represent the diminishing impact of increasing infectives on transmission.

The interaction of these two phenomena—behavioral modification and saturation—introduces rich nonlinear dynamics that significantly alter threshold values, equilibrium structures, and disease persistence conditions. However, despite their real-world relevance, most deterministic models analyze these mechanisms independently. This study addresses this gap by developing a comprehensive deterministic epidemic model incorporating both behavioral and saturated incidence mechanisms, followed by a rigorous mathematical and numerical analysis.

2. Objectives

This research aims to develop a comprehensive analytical framework for studying complex epidemic dynamics governed by nonlinear incidence. The specific objectives are as follows:

1. **Model Development:**
   Construct deterministic epidemic models (SIR/SEIR/SIQR type) incorporating both behavioral and saturated incidence functions.
2. **Basic Mathematical Properties:**
   Establish positivity, boundedness, and invariance of solutions within biologically meaningful regions.

3. **Equilibrium and Threshold Analysis:**
   Derive disease-free and endemic equilibrium points and determine the corresponding basic reproduction number R_0.
4. **Local Stability Analysis:**
   Utilize linearization, Jacobian matrices, and eigenvalue criteria to study the local stability of equilibrium points.
5. **Global Stability Analysis:**
   Use Lyapunov functions, LaSalle's Invariance Principle, or geometric approaches to establish global stability conditions where applicable.
6. **Bifurcation Analysis:**
   Identify and analyze forward/backward bifurcation phenomena and other qualitative transitions induced by nonlinear incidence.
7. **Parameter Sensitivity:**
   Determine the influence of behavioral and saturation parameters on key epidemic indicators.
8. **Numerical Simulation:**
   Perform simulations to illustrate model behaviour, validate theoretical results and visualize parameter-driven transitions.

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3. Methodology

The methodology includes analytical, numerical, and qualitative components:

Model Formulation

- Models of the SIR or SEIR type will be constructed with:
  - Behavioral incidence: $\frac{\beta SI}{1+\alpha I}$
  - Saturated incidence: $\frac{\beta SI}{1+kS}$
- Combined incidence may take forms such as:

$$\lambda(S,I) = \frac{\beta SI}{(1+\alpha I)(1+kS)}$$

- Parameter interpretations will be clearly defined.

Qualitative Analysis

- Positivity and biological feasibility of solutions will be established.
- Jacobian matrices will be derived and analyzed for local stability.
- The next-generation matrix approach will be used to compute R_0.
- Global stability will be attempted using:

- Lyapunov functions
- Bendixson-Dulac criteria
- Differential inequalities

Bifurcation and Sensitivity

- Routh–Hurwitz conditions will identify bifurcation thresholds.
- Numerical continuation tools may identify backward bifurcation, saddle-node bifurcation, etc.
- Sensitivity analysis will determine influence of α, k, and β.

Numerical Simulation

- MATLAB or Python will be used for:
  - Time series plots
  - Phase portraits
  - Parameter-sweep experiments
  - Bifurcation diagrams

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4 - Chapter wise Details:

CHAPTER 1: Introduction and Literature Review
CHAPTER 2-4: Model Formulation and Preliminary Concepts, Equilibrium Analysis and Stability , Bifurcation, Numerical Simulations and Validation
CHAPTER 5: Summary, Conclusions, and Future Directions

4 . Expected Outcomes

This work is expected to lead to the following outcomes:

1. A new deterministic epidemic model capturing behavioral and saturation effects in a unified way.
2. Analytical conditions for disease persistence or elimination based on nonlinear incidence parameters.
3. Identification of complex dynamical behaviour, including backward bifurcation and multiple endemic equilibria.
4. Demonstration that behavioral responses significantly lower the effective reproduction number.
5. Insight into how saturation contributes to peak infection reduction and stabilizes dynamics.
6. Numerical simulations that reinforce theoretical findings.

References:

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